

Measuring spin of a supermassive black hole at the Galactic Centre – Implications for a unique spin

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ABSTRACT

We determine the spin of a supermassive black hole in the context of disc seismology by comparing newly detected quasi-periodic oscillations (QPOs) of radio emission in the Galactic centre, Sagittarius A* (Sgr A*), as well as infrared and X-ray emissions with those of the Galactic black holes. We find that the spin parameters of black holes in Sgr A* and in Galactic X-ray sources have a unique value of ≈ 0.44 which is smaller than the generally accepted value for supermassive black holes, suggesting evidence for the angular momentum extraction of black holes during the growth of supermassive black holes. Our results demonstrate that the spin parameter approaches the equilibrium value where spin-up via accretion is balanced by spin-down via the Blandford-Znajek mechanism regardless of its initial spin. We anticipate that measuring the spin of black holes by using QPOs will open a new window for exploring the evolution of black holes in the Universe.

Key words: accretion, accretion discs – black hole physics – binaries: general – Galaxy: centre.

1 INTRODUCTION

The Galactic centre, Sagittarius A* (Sgr A*), is a compact source of radio, infrared, and X-ray emissions having variability in the range of a few tens of minutes to hours (Baganoff et al. 2001; Genzel et al. 2003; Yusef-Zadeh et al. 2006). These emissions seem to originate from a hot and low-density accreting gas plunging into a supermassive black hole (Yuan et al. 2004; Kato et al. 2009). A precise measurement of its mass and spin is a long-standing issue for astrophysics to investigate the mechanism of energy extraction from spinning black holes for astrophysical jet production as well as the evolution of supermassive black holes along the cosmic history (Bardeen 1970; Blandford & Znajek 1977; Wilson & Cobert 1995). Although the mass of Sgr A* has been constrained by using the stellar orbit method, a precise measurement of its spin for the best-estimated mass has been poorly conducted.

Recently, Miyoshi and colleagues have detected multiple quasi-periodic oscillations (QPOs) of radio emissions in Sgr A* (Miyoshi et al. in prep.), whose periods are close to the Keplerian period at the innermost stable circular orbit

of a supermassive black hole with mass $4 \times 10^6 M_\odot$. Because of the excellent spatial resolution of the Very Long Baseline Array (VLBA), the quasi-periodic radio emission certainly originates from within the central sub-mas scale, approximately $100 r_g$ around the central black hole at a distance of 7.6 kpc, where $r_g = GM/c^2 = 0.01 (M/10^6 M_\odot)$ AU is the gravitational radius (G , M , and c are the gravitational constant, the mass of black hole, and the speed of light, respectively). This is the first time that such multiple QPOs have been identified in the vicinity of a supermassive black hole. The spatial pattern of emission regions cannot be explained by the Keplerian rotation of a single emitting body at a given radius.

Four simultaneous QPOs (16.8, 22.2, 31.4, and 56.4 min) are detected and the first three periods are identical to QPOs in the near infrared and X-ray observations during different observation epochs (see Table 1). Three identical periods in the different wavelength are stable at least for several years and the frequency ratio of last two periods is close to 3:2. Such a stable double peak QPO is a well-known feature for high-frequency QPOs (HF-QPOs) in Galactic X-ray sources (Remillard & McClintock 2006). The multiple periodicity and their coincidence between the different wavelengths, and also the different observation epochs, indicates that the origin of QPOs in the Galactic centre is closely re-

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lated to the dynamics of an accretion disc feeding the black hole. Therefore we measure the spin parameter of a black hole in Sgr A* by using the period of QPOs based on disc-seismology (e.g., Nowak & Wagoner 1993).

2 METHOD AND MODEL

One promising mechanism of generating multiple QPOs is a global disc oscillation excited by the resonance between geodesic modes of the disc (the so-called *resonant disc oscillation* model: Abramowicz & Kluźniak 2001; Kato & Fukue 2006; Kato et al. 2008). The resonant frequency is the combination among geodesic frequencies at the radius where the resonance occurs. When the resonance condition is specified, both the resonant frequency and the resonant radius is determined uniquely in terms of the black hole mass M and the spin parameter $a_* \equiv Jc/GM^2$ where J is the angular momentum of the black hole. Therefore the metric of the black hole can be constrained by the frequency of the QPOs.

Resonance may occur at a radius where the frequency ratio of the geodesic modes is a ratio of small integers and resonant response can either spontaneously grow or damp the oscillation itself (Abramowicz & Kluźniak 2001). One of the most prominent resonances is a mode-coupling between acoustic waves and non-axisymmetric modes such as a warp in the disc, the so-called wave-warp resonance (Kato & Fukue 2006; Kato et al. 2008). For example, this resonance is excited at a radius r_{res} where $\Omega_K = 2\kappa$. Here Ω_K and κ are the Kepler frequency and the epicyclic frequency, respectively (see Fig. 1 of Kato & Fukue 2006 for the relation between a_* and r_{res}). Ω_K and κ at the resonant radius $\tilde{r}_{\text{res}} = r_{\text{res}}/r_g$ measured at infinity are expressed as

$$\Omega_K = \sqrt{\frac{GM}{r_{\text{res}}^3}} \left[1 + \frac{a_*}{\tilde{r}_{\text{res}}^{3/2}} \right]^{-1} \quad (1)$$

and

$$\kappa = \sqrt{\frac{GM}{r_{\text{res}}^3}} \frac{\sqrt{1 - 3/\tilde{r}_{\text{res}} + 8a_*(2\tilde{r}_{\text{res}})^{-3/2} - 3a_*^2(2\tilde{r}_{\text{res}})^{-2}}}{1 + a_*(2\tilde{r}_{\text{res}})^{-3/2}} \quad (2)$$

as derived by Okazaki et al. (1987). The resulting frequencies of QPOs are $m\Omega_K \pm \kappa$ and $m\Omega_K$ where m is the azimuthal mode number, and some lower mode oscillations related to such resonances are reported by numerical studies (Kato 2004).

3 RESULTS

3.1 Unified model of QPOs

Figure 1 shows the period of the observed QPO overlaid with lower mode ($m = 1, 2$) resonant periods related to the wave-warp resonance as a function of the black hole mass ranging from a stellar mass black hole to a supermassive black hole (skipping over the intermediate mass region). QPOs in the Galactic centre are selected with regard to the multiple detection among different wavelengths (Table 1). We found that such QPOs in Sgr A* detected at identical frequencies are consistent with a mass-period relation for the spin parameter $a_* \sim 0.4$ (see Fig. 1b). At the same time, HF-QPOs in the Galactic X-ray sources agree well with the

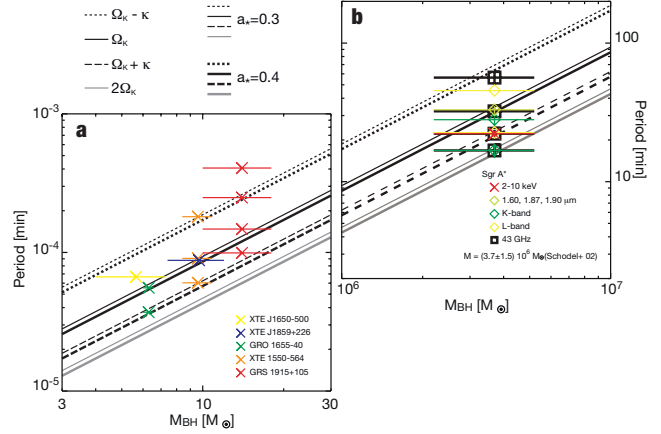


Figure 1. Observed QPO periods as a function of black hole mass. (a) HF-QPO periods of different sources are shown as crosses with horizontal bars indicating the range of black hole mass (Abramowicz & Kluźniak 2001; Homan et al. 2003; Orosz et al. 2004; Remillard & McClintock 2006; Kato et al. 2008). (b) QPO periods of Sgr A* in different energy bands are shown (see Table 1). The black hole mass is assumed to be $(3.7 \pm 1.5) \times 10^6 M_\odot$ (Schödel et al. 2002). Resonant oscillations for $m = 1$ and 2 are shown as solid (Ω_K), dashed ($\Omega_K + \kappa$), dotted ($\Omega_K - \kappa$), and gray solid ($2\Omega_K$) lines. Note that $2\Omega_K - \kappa = \Omega_K + \kappa$ and $2\Omega_K + \kappa$ are omitted for simplicity. Thin and thick lines indicate the periods for the spin parameter $a_* = 0.3$ and 0.4, respectively.

resonant periods for the same spin parameter within the error of the estimated mass (Fig. 1a). Therefore we identify the three identical periods (16.8, 22.2, and 31.4 min) with resonant modes $2\Omega_K$, $\Omega_K + \kappa$, and Ω_K , respectively.

3.2 Unique spin parameter

Now we can determine the spin parameter of black holes by using the periods of QPOs corresponding to Ω_K . For instance, 31.4 min is used for Sgr A* and periods of lower HF-QPOs are used for the Galactic X-ray sources. Note that the frequency of single peak HF-QPOs are treated as Ω_K . In order to constrain the resultant spin parameter, the estimated mass of a supermassive black hole in Sgr A* is taken from recent measurements (Schödel et al. 2002; Ghez et al. 2008; Gillessen et al. 2009). Figure 2 shows spin parameters of all samples evaluated by using the discseismic measurement. All spin parameters are relatively small (≤ 0.7) in comparison with the equilibrium value of spinning black holes (≈ 0.95) predicted by a numerical study (Gammie et al. 2004). When all samples are fitted by using a linear relation as a function of the black hole mass, the spin parameter becomes larger than 1 for black holes with $M \geq 10^7 M_\odot$. Instead of a linear relation, we obtain a best-fit unique spin parameter $a_* = 0.44 \pm 0.08$, which is depicted by a gray shaded region, for 1σ uncertainty by linear least square fitting.

3.3 Evolution of BH spin and mass

Next, we should ask why black holes have a unique spin parameter in spite of the fact that their age as well as mass accretion history may vary in general. Actually, our results contradict recent studies that predict extremely spinning

Table 1. QPOs detected in Sgr A*.

Obs. epoch (UT)	Obs. band	Period (min)	Ref. #
2003/06/15 - 16	K-band	$16.8 \pm 2, 28.0$	Genzel et al. 2003
2004/09	1.60, 1.87, 1.90 (μm)	33 ± 2	Yusef-Zadeh et al. 2006
2002/10, 2004.08	2 – 10 (keV)	22.2	Bélanger et al. 2006
2007/04/04	L-band	22.6	Hamaus et al. 2009
2007/07/22	L-band	45.4	Hamaus et al. 2009
2004/03/08 09:30 - 16:30	43 (GHz)	$16.8 \pm 1.4, 22.2 \pm 1.4, 31.2 \pm 1.5, 56.4 \pm 6$	Miyoshi et al. in prep.

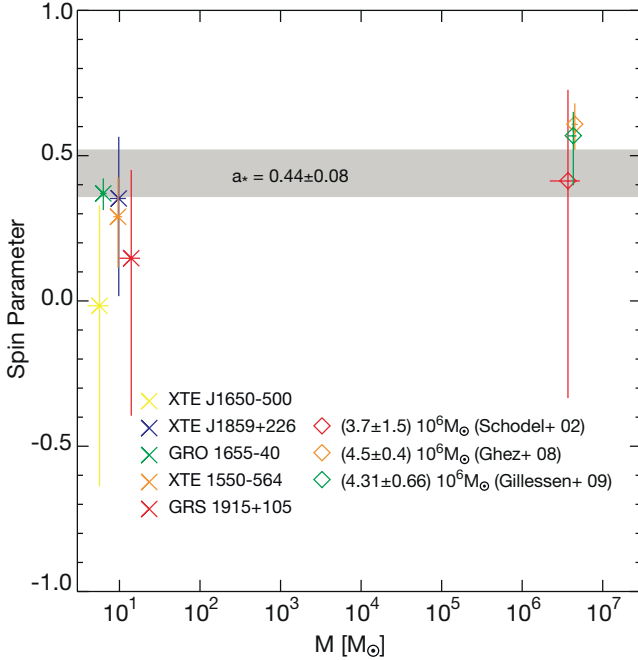


Figure 2. Spin parameters measured by discseismic method. Crosses indicate spins for the Galactic X-ray sources whereas diamonds indicate those for the Galactic centre in terms of black hole masses measured by the stellar orbits method (Schödel et al. 2002; Ghez et al. 2008; Gillessen et al. 2009). A gray shaded region indicate the best-fit spin parameter $a_* = 0.44 \pm 0.08$ for 1σ uncertainty.

black holes (Shapiro 2005; Volonteri et al. 2005). In order to test the feasibility of such a small unique spin parameter, we have to study the spin-up process by mass accretion and the spin-down process by the energy extraction as a result of the Blandford-Znajek mechanism, simultaneously.

Figure 3 represents the equilibrium value of spin and also the time evolution of black holes surrounded by a relativistic standard accretion disc (Novikov & Thorne 1973; Page & Thorne 1974; see also Kato et al. 2008), assuming given disc parameters such as the viscosity parameter α (Shakura & Sunyaev 1973), the magnetized parameter β , the ratio of the gas pressure to the magnetic pressure, and the mass accretion rate $\dot{m} = \dot{M}/\dot{M}_{\text{EDD}}$ normalized by the Eddington mass accretion rates $\dot{M}_{\text{EDD}} = 4\pi GM/c\kappa_{\text{es}}$ where κ_{es} is the electron scattering opacity (e.g., Kato et al. 2008). In general, these parameters are not independent because magnetohydrodynamic (MHD) turbulence in the disc is thought to be the source of viscosity and their values can only be examined numerically. For instance, we employ $\alpha = 0.01$ on

the basis of three-dimensional MHD simulations showing the total stress corresponds to $\alpha \approx 0.02 - 0.06$ (Hawley 2000; Machida et al. 2000) for $\dot{m} \ll 1$ and $\alpha \approx 0.01$ for $\dot{m} \sim 1$ (Hirose et al. 2006). Recent MHD simulations also exhibit the natural emergence of large-scale magnetic fields (the so-called magnetic tower) at the inner region of an accretion disc (Kato et al. 2004). The formation of a magnetic tower is key to the extraction of the energy and angular momentum of a spinning black hole by the Blandford-Znajek mechanism and it has been suggested that the necessary condition for the energy and angular momentum extraction at the innermost region of an accretion disk is $\beta \approx 1$ (McKinney & Gammie 2004).

The equations we solved in this study are the followings:

$$\frac{d \ln M}{dt} = \frac{\dot{M}}{M} e_{\text{in}} - \frac{\mathcal{P}}{Mc^2} \quad (3)$$

$$\frac{dJ}{dt} = \dot{M} l_{\text{in}} - \frac{\mathcal{P}}{\Omega_F} \quad (4)$$

where \dot{M} , e_{in} , and l_{in} are the mass accretion rate, the specific energy and the specific angular momentum at the inner edge of the accretion disc, respectively. The electromagnetic power loss \mathcal{P} from the black hole is assumed to be that of the Blandford-Znajek mechanism:

$$\mathcal{P} = \mathcal{P}_{\text{BZ}} \simeq \frac{1}{8} \frac{B_{\perp}^2 r_{\text{H}}^4}{c} \Omega_F (\Omega_{\text{H}} - \Omega_F) \quad (5)$$

where r_{H} is the radius of the event horizon and Ω_F and Ω_{H} are the angular velocity of the magnetic fields permeating the horizon and the angular velocity of the black hole, respectively (see Moderski & Sikora 1996; Beskin et al. 2003). The strength of magnetic fields B_{\perp} permeating the event horizon is assumed to be regulated by the pressure of accretion disc p_{disc} so that $B_{\perp}^2 = 8\pi p_{\text{disc}}/\beta$. Note that the electromagnetic power loss is not negligible when β is less than the order of the unity.

The relativistic standard accretion disc model provides a complete set of equations for describing the pressure of accretion disc at the given radius as a function of the viscosity parameter α , the black hole mass $m = M/M_{\odot}$, the spin parameter a_* , and the mass accretion rate \dot{m} . For a given \dot{m} , the radiation pressure dominated region appears within the radius:

$$\begin{aligned} \tilde{r}_{\text{b}} &= r_{\text{b}}/r_{\text{g}} \\ &= 36\alpha^{2/21} m^{2/21} \dot{m}^{16/21} \mathcal{B}^{-16/21} \mathcal{D}^{2/21} \mathcal{H}^{-10/21} \mathcal{Q}^{16/21} \end{aligned} \quad (6)$$

where \mathcal{B} , \mathcal{D} , \mathcal{H} , and \mathcal{Q} are the general relativistic correction factors (Page & Thorne 1974). To summarize, the pressure of the accretion disc can be described as follows:

$$p_{\text{disc}} = \begin{cases} p_{\text{rad}} & \tilde{r} \leq \tilde{r}_b, \\ p_{\text{gas}} & \tilde{r} > \tilde{r}_b, \end{cases} \quad (7)$$

and

$$p_{\text{rad}} = 1.4 \times 10^{16} (\alpha m)^{-1} \mathcal{R}_1 \text{ dyne cm}^{-2}, \quad (8)$$

$$p_{\text{gas}} = 3.0 \times 10^{17} (\alpha m)^{-9/10} \dot{m}^{4/5} \mathcal{R}_2 \text{ dyne cm}^{-2}, \quad (9)$$

where $\mathcal{R}_1 = \tilde{r}^{-3/2} \mathcal{B}^{-2} \mathcal{D}^{-1} \mathcal{C}$ and $\mathcal{R}_2 = \tilde{r}^{-51/20} \mathcal{B}^{-14/5} \mathcal{D}^{-9/10} \mathcal{C} \mathcal{H}^{-1/2} \mathcal{Q}^{4/5}$ are the radial dependence including the general relativistic correction factors at the Boyer-Lindquist coordinated radius $\tilde{r} = c^2 r / GM$. The radius for evaluating the strength of magnetic field is assumed to be $\tilde{r}_0 = 1.3 \tilde{r}_{\text{ms}}$ where \tilde{r}_{ms} is the marginally stable circular orbit (Bardeen et al. 1972):

$$\tilde{r}_{\text{ms}} = 3 + z_2 - \{(3 - z_1)(3 + z_1 + 2z_2)\}^{1/2}, \quad (10)$$

where

$$z_1 = 1 + (1 - a_*^2)^{1/3} [(1 + a_*)^{1/3} + (1 - a_*)^{1/3}] \quad (11)$$

$$z_2 = (3a_*^2 + z_1^2)^{1/2}. \quad (12)$$

Finally, we rewrite the equation (3) & (4) by using the normalized variables as

$$\frac{d \ln m}{dt} = \frac{1}{\tau_{\text{EDD}}} (\tilde{e}_{\text{in}} - \eta_{\text{BZ}}), \quad (13)$$

$$\frac{da_*}{dt} = \frac{1}{\tau_{\text{EDD}}} \left[(\tilde{l}_{\text{in}} - 2a_* \tilde{e}_{\text{in}}) - 2\eta_{\text{BZ}} \left(\frac{\tilde{r}_H}{ka_*} - a_* \right) \right], \quad (14)$$

where symbols are the Eddington time $\tau_{\text{EDD}} = M/\dot{M}_{\text{EDD}}$, the specific energy input $\tilde{e}_{\text{in}} = e_{\text{in}}/c^2$, the efficiency of the Blandford-Znajek mechanism $\eta_{\text{BZ}} = \mathcal{P}_{\text{BZ}}/\dot{M}_{\text{EDD}} c^2$, the specific angular momentum input $\tilde{l}_{\text{in}} = cl_{\text{in}}/GM$, the horizon radius $\tilde{r}_H = c^2 r_H / GM = 1 + (1 - a_*^2)^{1/2}$, and $k = \Omega_F / \Omega_H = 1/2$ for the maximum efficiency of the Blandford & Znajek mechanism. Here we assume that the inner boundary is at the marginally stable circular orbit and both the energy and the angular momentum of accreting matter at the boundary are advected into the black hole. The specific energy and the specific angular momentum at the boundary are:

$$\tilde{e}_{\text{in}} = \tilde{e}_{\text{ms}} = \sqrt{1 - \frac{2}{3\tilde{r}_{\text{ms}}}}, \quad (15)$$

$$\tilde{l}_{\text{in}} = \tilde{l}_{\text{ms}} = 2\sqrt{3} \left(1 - \frac{2a_*}{3\sqrt{\tilde{r}_{\text{ms}}}} \right). \quad (16)$$

We numerically integrated equations (13) & (14) with given initial parameters and track the evolution of black hole mass and spin. We also determined the equilibrium spin for $m = 10, 10^6, 10^8$ by solving $da_*/dt = 0$ in the equation (14) by using bisection method.

Figure 3a shows the equilibrium value of spin as a function of $\alpha \dot{m}$. The equilibrium spin becomes larger when either α or \dot{m} becomes larger. The best-fit spin parameter determined by the discseismic method corresponds to an equilibrium value of $\dot{m} \approx 1$. Figure 3b shows the time evolution of spin, where the spin parameter of each model converges to a unique value regardless of the initial one. When the mass accretion rate is regulated by the Eddington value ($\dot{m} = 1$), the spin converges to the equilibrium value ≈ 0.55 for stellar-mass black holes within the order of 10^8 years and then slowly approaches the equilibrium value ≈ 0.4 for massive

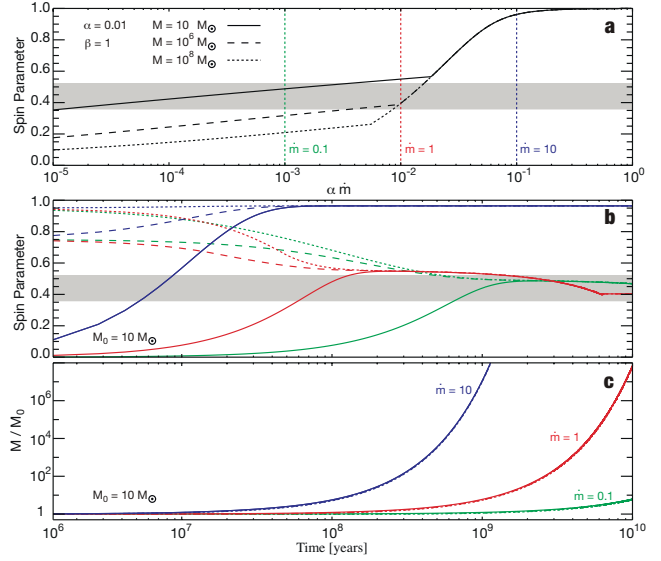


Figure 3. Time evolution of the black hole surrounded by the standard accretion disc with the suitable disc parameter $\alpha = 0.01$ and $\beta = 1$ for different mass accretion rates ($\dot{m} = 0.1, 1.0$, and 10 denoted by a green, red, and blue line, respectively). (a) is the equilibrium spin parameter in terms of $\alpha \dot{m}$ for given black hole masses ($M = 10, 10^6, 10^8 M_\odot$ denoted by a black solid, dashed, and dotted line, respectively). (b) is the time evolution of spin parameter for the initial black hole mass $M_0 = 10 M_\odot$ with different initial spin parameters ($a_* = 0.0, 0.75$, and 0.95 denoted by a solid, dashed, and dotted line, respectively). A gray shaded region in (a) and (b) indicates the best-fit spin parameter $a_* = 0.44 \pm 0.08$ determined by the discseismic measurement. (c) is the time evolution of mass ratio M/M_0 with different mass accretion rate and initial spin parameters. The curves are almost independent of the initial spin parameters.

black holes. When $\dot{m} = 0.1$, the spin converges to a value ≈ 0.5 within the hubble time, but never actually reaches the equilibrium spin. Therefore the resultant spin is consistent with the small unique spin ≈ 0.44 when the mass accretion rate is regulated by the Eddington value $\dot{m} \sim 1$ with the appropriate disc parameters. On the other hand, when the accretion disc is somehow in a super-critically accreting phase, with $\dot{m} = 10$, the spin converges to the equilibrium value of ≈ 0.96 within the order of 10^7 years. Although the equilibrium spin of the super-critical accretion phase is larger than the unique value, it could approach to this value during the subsequent sub-critical accretion phase in less than 10^9 years. The evolution of the black hole mass is not affected by the initial spin parameter (see Fig. 3c). Note that the final mass becomes 10^6 times larger than the initial mass for $\dot{m} \geq 1$.

4 CONCLUSIONS

It has been suggested that the supermassive black hole in the Galactic centre used to be in the nearly critical mass accretion phase for more than the order of 10^8 years. A possible explanation for such a large mass accretion history is the massive star formations in the proximity of the Galactic centre region. During the critical accretion phase, the spin reaches the unique value and the mass becomes $\sim 10^6 M_\odot$,

which is then maintained during the subsequent low accretion rate phase. Note that stochastic mass accretion history may also help to create the moderately spinning massive black hole (King & Pringle 2006). Similarly, black holes in Galactic X-ray sources have been in the nearly critical accretion rate phase for order 10^8 years as well, suggesting their companion stars should be low-mass stars. Because they have reached the quasi-equilibrium state, the limit-cycle activities and also the emergence of jets does not alter their spin evolution. Thus, we conclude that the spin parameter of a supermassive black hole in the Galactic centre has a unique value of $a_* = 0.44 \pm 0.08$. Conversely, the mass of a black hole consistent with the unique spin is $M = (4.2 \pm 0.4) \times 10^6 M_\odot$.

Without detecting the event horizon, we have constrained the mass and spin of the supermassive black hole at the Galactic centre. The method we used here depends entirely on geodesic frequencies that are independent of the distance and viewing angle of a black hole. Once the unique spin parameter of the black hole in the Galactic centre has been confirmed by detection of the event horizon in the future observations (e.g., Takahashi 2004), studies of QPOs in other galaxies will open a new window to survey the growth history of massive black holes (Markowitz et al. 2007; Gierliński et al. 2008).

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REFERENCES

- Abramowicz, M. A., & Kluźniak, W. 2001, *A&Ap*, 374, L19
 Baganoff, F. K., et al. 2001, *Nature*, 413, 45
 Bardeen, J. M. 1970, *Nature*, 226, 64
 Blandford, R. D., & Znajek, R. L. 1977, *MNRAS*, 179, 433
 Bélanger, G., Terrier, R., de Jager, O. C., Goldwurm, A., & Melia, F. 2006, *Journal of Physics Conference Series*, 54, 420
 Beskin, V. et al. 2003, *Accretion disks, jets and high-energy phenomena in astrophysics*, Ecole d'été de Physique des Houches, Session LXXVIII, July 29-August 23, 2002, NATO Advanced Study Institute, Euro Summer School, Ecole thematique du CNRS. Edited by V. Beskin, G. Henri, F. Menard, G. Pelletier, and J. Dalibard. Heidelberg: Springer, 2003.,
 Gammie, C. F., Shapiro, S. L., & McKinney, J. C. 2004, *ApJ*, 602, 312
 Genzel, R., Schödel, R., Ott, T., Eckart, A., Alexander, T., Lacombe, F., Rouan, D., & Aschenbach, B. 2003, *Nature*, 425, 934
 Ghez, A. M., et al. 2008, *ApJ*, 689, 1044
 Gierliński, M., Middleton, M., Ward, M., & Done, C. 2008, *Nature*, 455, 369
 Gillessen, S., Eisenhauer, F., Trippe, S., Alexander, T., Genzel, R., Martins, F., & Ott, T. 2009, *ApJ*, 692, 1075
 Hamaus, N., Paumard, T., Müller, T., Gillessen, S., Eisenhauer, F., Trippe, S., & Genzel, R. 2009, *ApJ*, 692, 902
 Hawley, J. F. 2000, *ApJ*, 528, 462
 Hirose, S., Krolik, J. H., & Stone, J. M. 2006, *ApJ*, 640, 901
 Homan, J., Klein-Wolt, M., Rossi, S., Miller, J. M., Wijmands, R., Belloni, T., van der Klis, M., & Lewin, W. H. G. 2003, *ApJ*, 586, 1262
 Kato, S., & Fukue, J. 2006, *PASJ*, 58, 909
 Kato, S., Fukue, J., & Mineshige, S. 2008, *Black-Hole Accretion Disks — Towards a New Paradigm* —, 549 pages, including 12 Chapters, 9 Appendices, ISBN 978-4-87698-740-5, Kyoto University Press (Kyoto, Japan), 2008.
 Kato, Y. 2004, *PASJ*, 56, 931
 Kato, Y., M. Umemura, & K. Ohsuga 2009)
 Kato, Y., Umemura, M., & Ohsuga, K. 2009, *MNRAS*, 400, 1742
 King, A. R., & Pringle, J. E. 2006, *MNRAS*, 373, L90
 Machida, M., Hayashi, M. R., & Matsumoto, R. 2000, *ApJ*, 532, L67
 Markowitz, A., Papadakis, I., Arévalo, P., Turner, T. J., Miller, L., & Reeves, J. N. 2007, *ApJ*, 656, 116
 McKinney, J. C., & Gammie, C. F. 2004, *ApJ*, 611, 977
 Miyoshi, M. et al. in prep.
 Moderski, R., & Sikora, M. 1996, *MNRAS*, 283, 854
 Novikov, I. D., & Thorne, K. S. 1973, *Black Holes (Les Astres Occlus)*, 343
 Nowak, M. A., & Wagoner, R. V. 1993, *ApJ*, 418, 187
 Okazaki, A. T., Kato, S., & Fukue, J. 1987, *PASJ*, 39, 457
 Orosz, J. A., McClintock, J. E., Remillard, R. A., & Corbel, S. 2004, *ApJ*, 616, 376
 Remillard, R. A., & McClintock, J. E. 2006, *Annu. Rev. Astro. Astrophys.*, 44, 49
 Schödel, R., et al. 2002, *Nature*, 419, 694
 Shakura, N. I., & Sunyaev, R. A. 1973, *Astron. & Astrophysics.*, 24, 337
 Shapiro, S. L. 2005, *ApJ*, 620, 59
 Takahashi, R. 2004, *ApJ*, 611, 996
 Volonteri, M., Madau, P., Quataert, E., & Rees, M. J. 2005, *ApJ*, 620, 69
 Wilson, A. S., & Colbert, E. J. M. 1995, *ApJ*, 438, 62
 Yusef-Zadeh, F., et al. 2006, *ApJ*, 644, 198
 Yuan, F., Quataert, E., & Narayan, R. 2004, *ApJ*, 606, 894